Studio 8: Floating Ducks

This following is a diagram that describes the model used in the Matlab to calculate the depth at which the duck will float.

r=10cm

p=.3 g/cm^3

d

The equations and constants used are as follows, and are derived from fluid dynamics (and the equations provided by the instructor):

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in order to have object float, so:

Using these equations, we solve for d to find the depth at which the weight of the water displaced by the duck is equal to the weight of the entire duck, because at this depth the duck will float.

To see how this was translated into Matlab, see accompanying code. The software tools used were m-files, error functions, and the fzero command in Matlab. These tools allowed me to set up the above parameters and equations and then calculated when , which is when the duck would float.

Any casual observer can tell that this is only an approximation. A duck is not equivalent to a sphere, therefore the depth at which an actual duck would float will likely be different from the answer given by the above method. However, the geometry of a duck is close to that of a sphere, and when one sets the radius of the approximation, it is expected that the said radius is a reasonable estimate(average) of the overall duck. That is, since a duck is smaller on top than on the bottom, but circular throughout, an appropriate radius is the average of the smaller and larger radii.

In the original experiment, with the given parameters .3 and 10 cm, d is 7.27 cm. As density of the duck is decreased, the height decreases, which makes sense. If the volume of the duck remained the same but the mass per volume value decreased, there would be less “duck” for the water to support. Hence, less water needs to be displaced for the duck to float, and because the distribution of volume is the same, the height of water decreases as well. Conversely, when density was increased, height increased; the same principle applies here. If density exceeds 1, then the duck would appear to be submerged in water but still floating, according to the code (if allow d>2r). However, because the calculations assume there is volume above the sphere still displacing water, this is a weakness of the code.

When density remained constant and the radius of the duck increased, height decreased. This also makes physical sense because per cm of height, more water/weight is displaced, so less height is needed for the duck to float. If radius were to continue to increase, the height would continue to decrease proportional to the change in r. It does not make sense for r to be zero, but at values very close zero, the duck would essentially be submerged because very little water is displaced per cm of height.

Evidence that this simulation is correct includes the fact that it makes intuitive and physical sense, that the approximation of the geometry of a duck as a sphere is a valid one, and that classmates corroborated results.